Supplementary problem 1: Computing log-likelihood function and plotting it (Numerical problem) by R software.

Problem 1 Suppose that $X_1, \cdots, X_n$ form a random sample from a distribution for which the pdf $f(x|\theta)$ is as follows:

$$f(x|\theta) = \begin{cases} \theta x^{\theta-1}, & \text{for} \quad 0 < x < 1 \\ 0, & \text{for} \quad x \leq 0 \end{cases}$$

Also suppose that the value of θ is unknown ($\theta > 0$). Find the MLE of θ.

n=10,100,1000 generating samples from Beta(2,1)

Problem 2 Suppose that $X_1, \cdots, X_n$ form a random sample from a distribution for which the pdf $f(x|\theta)$ is as follows:

$$f(x|\theta) = \frac{1}{2} e^{-|x-\theta|} \quad \text{for} \quad -\infty < x < \infty$$

Also suppose that the value of θ is unknown ($-\infty < \theta < \infty$). Find the MLE of θ.

n= 10,100,1000 generating samples from Laplace distribution Laplace(10,1)

Problem 3 Suppose that $X_1, \cdots, X_n$ form a random sample from a distribution for which the pdf $f(x|\theta)$ is as follows:

$$f(x|\theta) = \begin{cases} e^{\theta-x}, & \text{for} \quad x > \theta \\ 0, & \text{for} \quad x \leq \theta \end{cases}$$

Also suppose that the value of θ is unknown ($-\infty < \theta < \infty$). a) Show that the MLE of θ does not exist. b) Determine another version of the pdf of this same distribution for which the MLE of θ will exist, and find this estimate.

n= 10 , 100, 1000 generating samples from N(0,5)